der Bundeswehr
Universität München

# Doppler Shift Based Opportunistic LEO-PNT With Starlink Signals

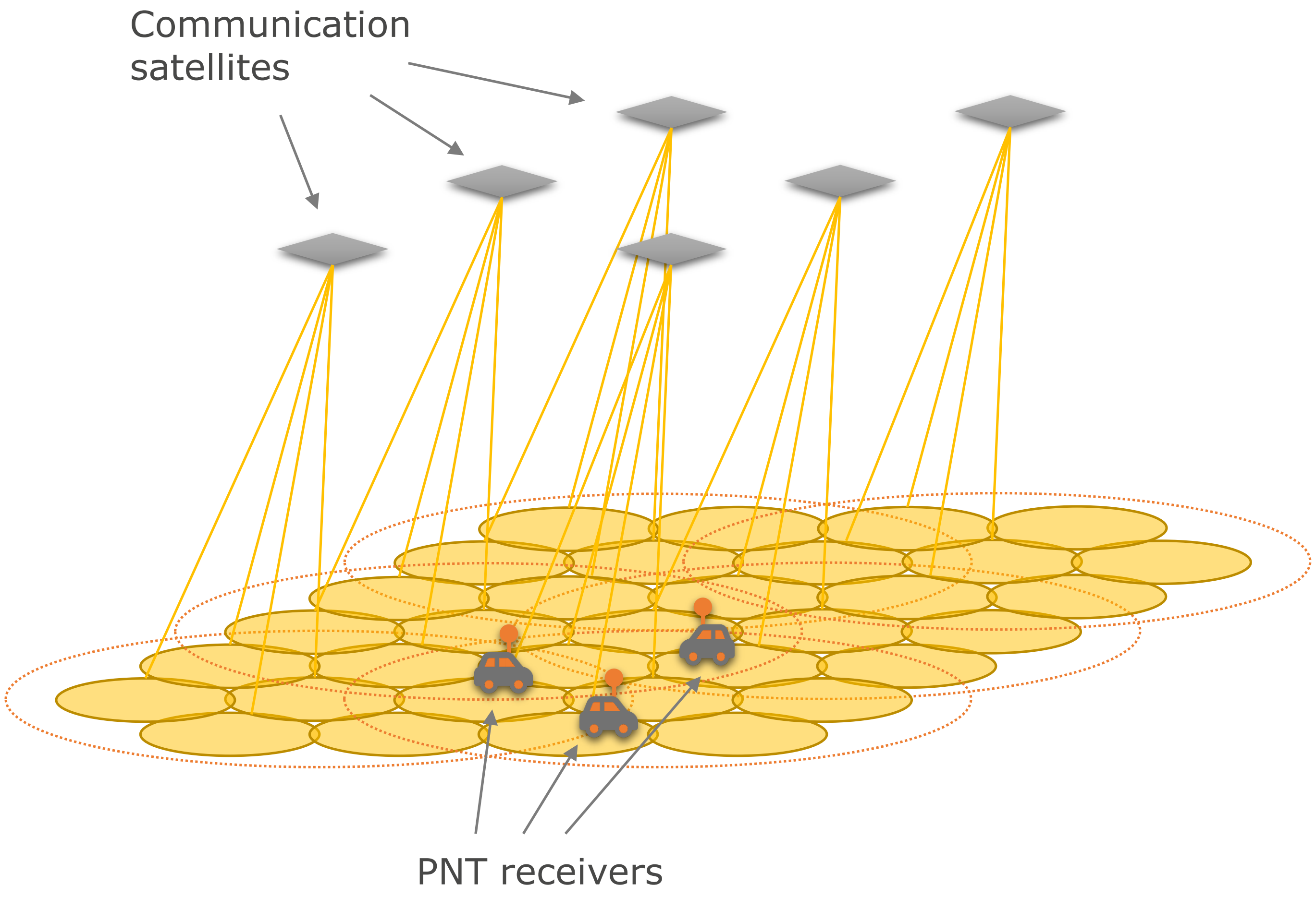
Winfried Stock

IEEE LEO Sats Workshop: LEO Position, Navigating, and Timing (PNT)
5 May 2025

Munich Center for Space Communications

Chair of Signal Processing
Univ.-Prof. Dr.-Ing. Andreas Knopp

# Opportunistic LEO-PNT




- (Unmodified) LEO satellites transmit communication signals
- PNT estimation at the receiver based on
  - Observation of the communication signals (as „signals of opportunity")
  - Satellite orbit

✓ No additional (PNT) satellites required

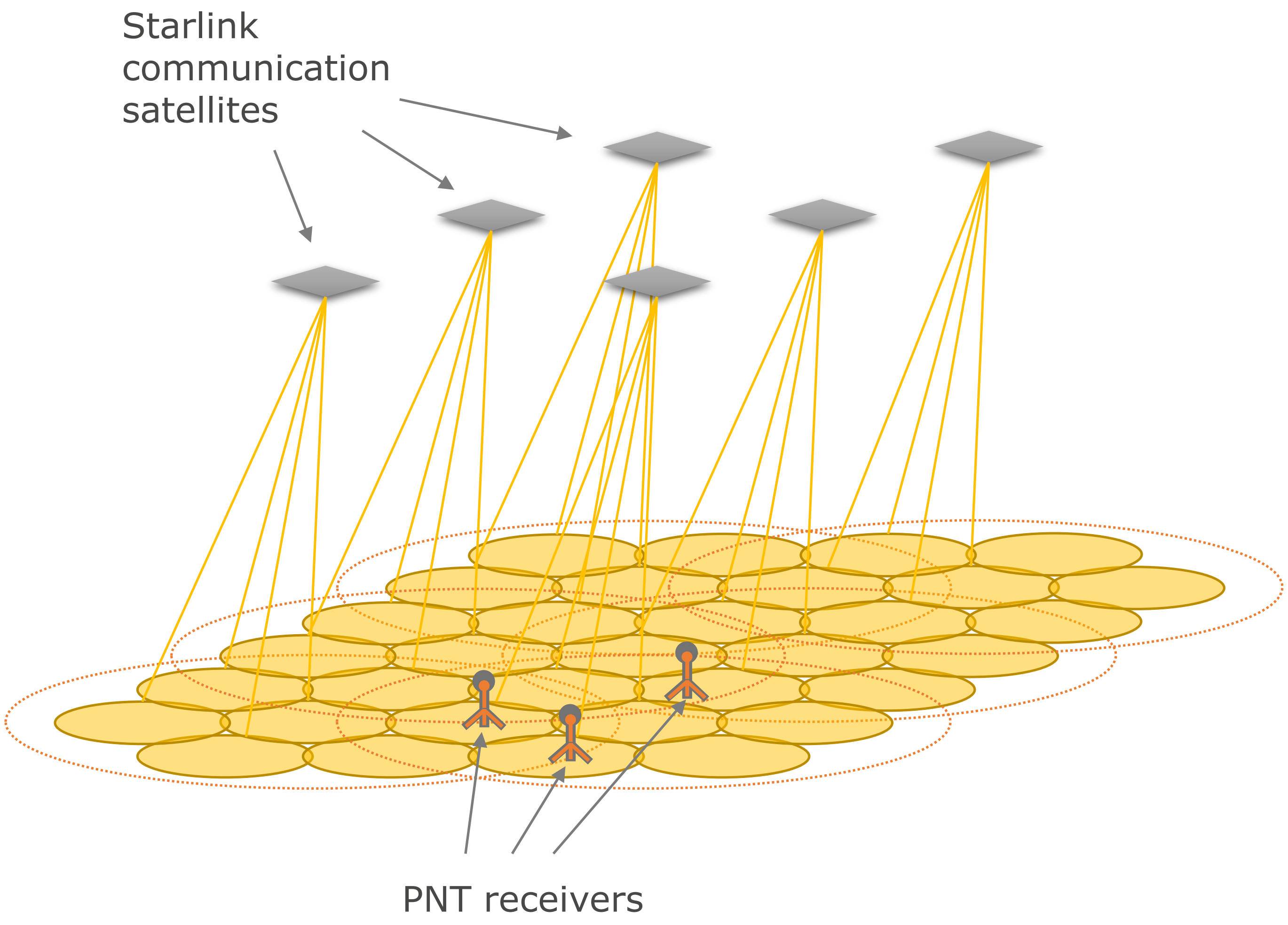
✓ Approach for complementary PNT in case of interruption or degradation of GNSS

✓ Global coverage

Which sources of error are (most) relevant?

W. Stock, R. T. Schwarz, and A. Knopp, “Error Source Analysis for Doppler Shift Based Opportunistic LEO-PNT With Starlink Signals,” 2025 IEEE/ION Position, Location and Navigation Symposium (PLANS), Salt Lake City, UT, April 2025.

# Investigated scenario




The investigated scenario consideres a

- stand-alone
- static
- opportunistic LEO-PNT receiver, performing
- multi-epoch
- Doppler-shift measurements of
- Starlink signals, leveraging
- consecutive Primary Synchronization Sequences (PSS).

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**Frequency estimation error**

Satellite orbit error

Clock errors
(Satellite and receiver)

Atmospheric errors

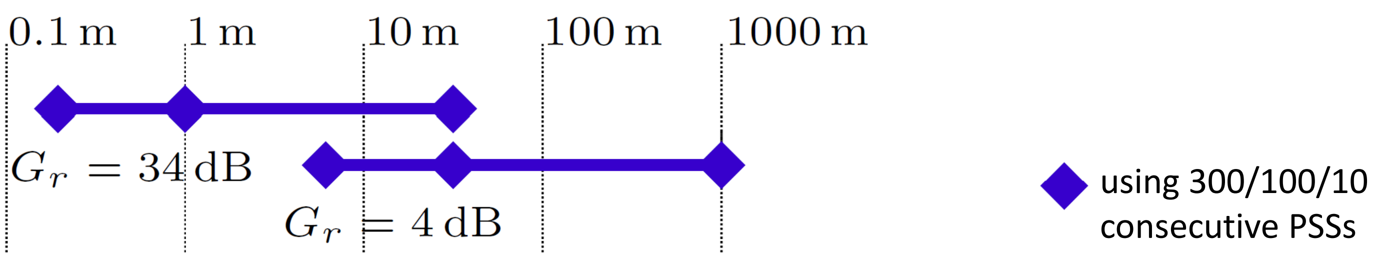
## Modified Cramér-Rao Bound

$$\mathrm{MCRB}(\nu) = \frac{3T_s}{(T_s L_0)^3 2\pi^2} \frac{1}{\mathrm{SNR}_r}$$

$$\Delta r = \frac{\lambda_c \cdot \Delta f}{\gamma}$$

$$\gamma = \frac{1}{1 - \frac{R_E}{a_{orb}}} \sqrt{\frac{\mu}{a_{orb}^3}}$$

## Impact on positioning accuracy:



Frequency estimation error

**Satellite orbit error**

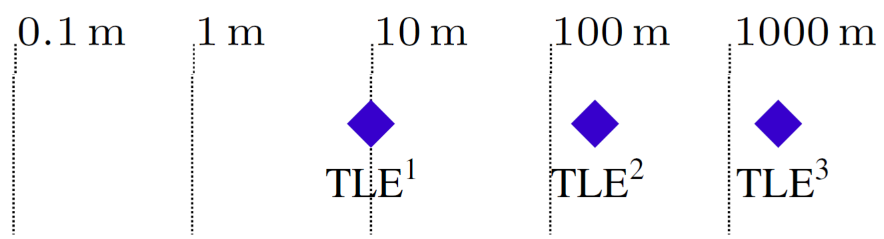
Clock errors
(Satellite and receiver)

Atmospheric errors

## Monte-Carlo-Simulation of a single satellite passover

| | **Scenario** | **Orbit error** along | cross | radial |
|---|---|---|---|---|
| 3 | NORAD TLE files | 2 km | 200 m | 200 m |
| 2 | adjusted TLE files | 200 m | 200 m | 200 m |
| 1 | precise TLE files | 10 m | 10 m | 10 m |

## Impact on positioning accuracy:



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Frequency estimation error

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## Short-term clock stability

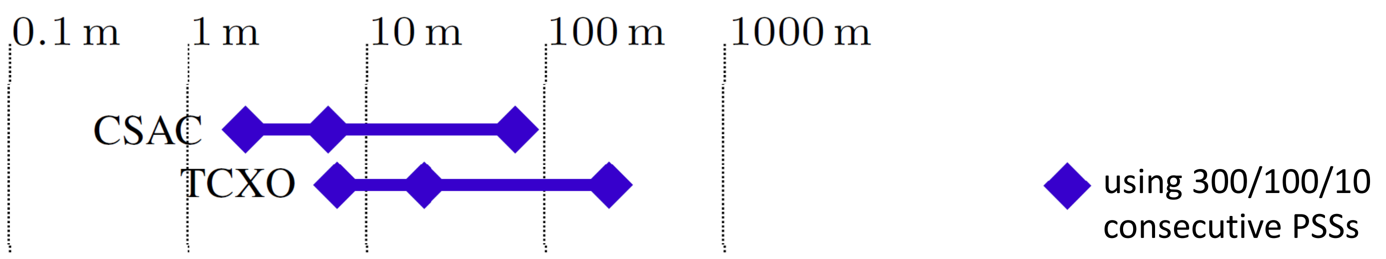
$$\sigma_{f,clock} = \frac{2\pi}{2.25 \cdot 2\pi T_c} \sigma_y(\tau)\tau f_c$$

relationship for a 3rd order PLL

## assumed Allan deviations

| **Clock** | $\sigma_y(\tau)$ |
|---|---|
| TCXO | $1 \cdot 10^{-9}\ \tau^{-\frac{1}{2}}$ |
| CSAC | $3 \cdot 10^{-10}\ \tau^{-\frac{1}{2}}$ |
| OCXO | $1 \cdot 10^{-12}$ |

## Impact on positioning accuracy:



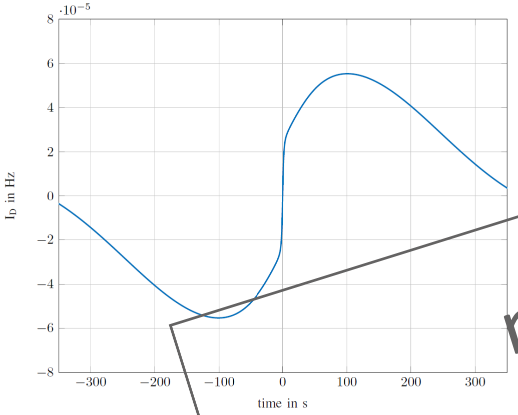
Frequency estimation error

Satellite orbit error

Clock errors
(Satellite and receiver)

**Atmospheric errors**

## Ionospheric delay rate




$$I_z = \frac{40.3 \cdot \mathrm{TEC}}{f_c^2}$$

$$I_\rho = OF_I(\xi) \cdot I_z$$

$$I_D = \frac{\partial I_\rho}{\partial t} \frac{f_c}{c}$$

negligible

## Tropospheric delay rate

- Literature (GNSS)

# Comparison

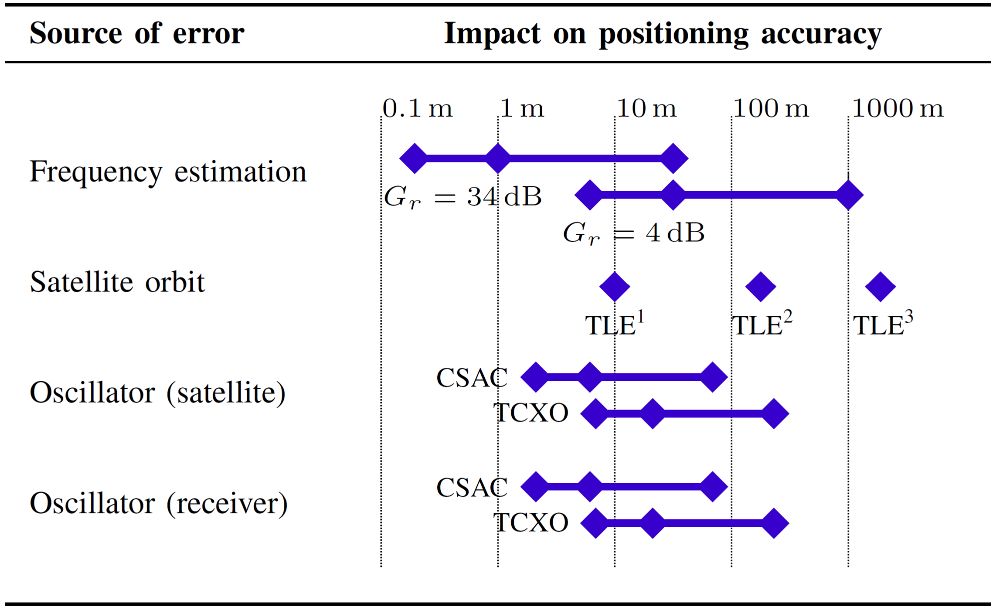
**Frequency estimation error**

**Satellite orbit error**

**Clock errors**
**(Satellite and receiver)**

Atmospheric errors

TABLE VI
IMPACT OF RELEVANT ERROR SOURCES ON THE POSITIONING ACCURACY FOR 300/100/10 PSSs / FRAMES USED PER MEASUREMENT



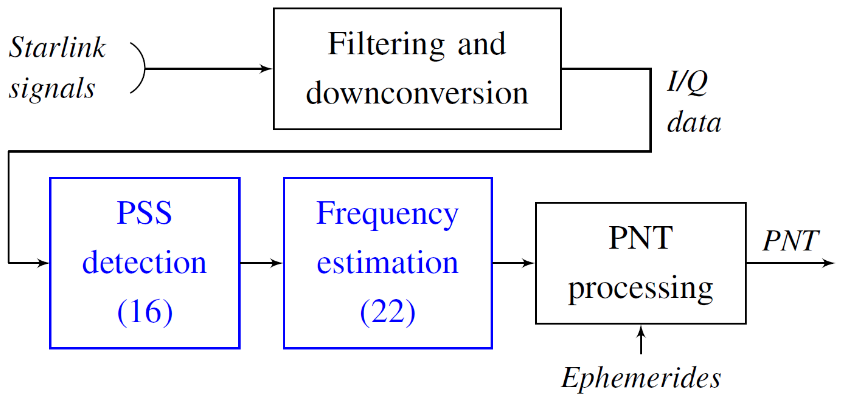

[1] Accurate TLE files
[2] Adjusted NORAD TLE files
[3] NORAD TLE files

How to estimate the frequency of the signal
most accurately
and
computationally efficient?

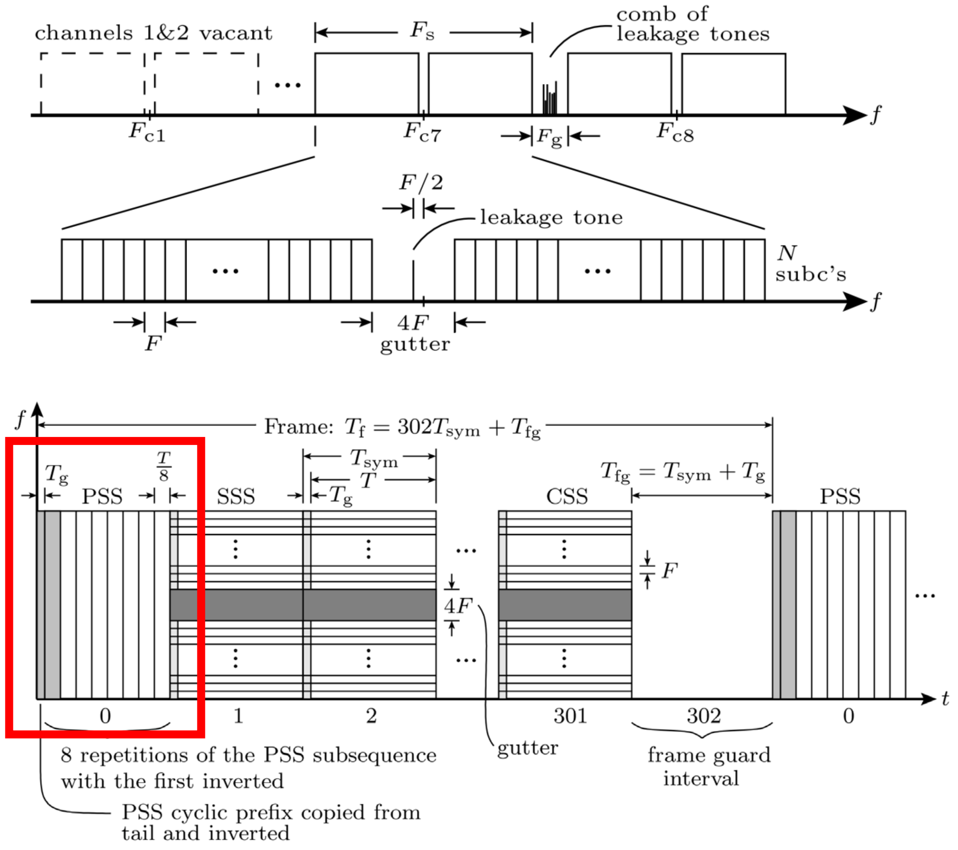
W. Stock, C. A. Hofmann and A. Knopp, "Correlation-Based Doppler Shift Estimation for Opportunistic LEO-PNT With Starlink Signals," 2024 IEEE International Conference on Wireless for Space and Extreme Environments (WiSEE), Daytona Beach, FL, USA, Dec. 2024.

# Scope

- Computationally efficient algorithm
- for Starlink Primary Synchronization Sequence (PSS)
- detection and frequency estimation
- for Doppler-shift based opportunistic LEO-PNT




**Starlink Primary Synchronization Sequence (PSS)**

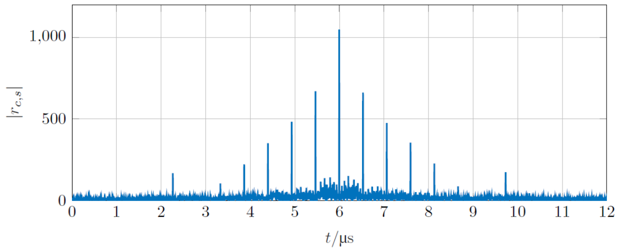



T. E. Humphreys, P. A. Iannucci, Z. M. Komodromos and A. M. Graff, "Signal Structure of the Starlink Ku-Band Downlink," in IEEE Transactions on Aerospace and Electronic Systems, vol. 59, no. 5, pp. 6016-6030, Oct. 2023, doi: 10.1109/TAES.2023.3268610.

# Correlation-based PSS detection

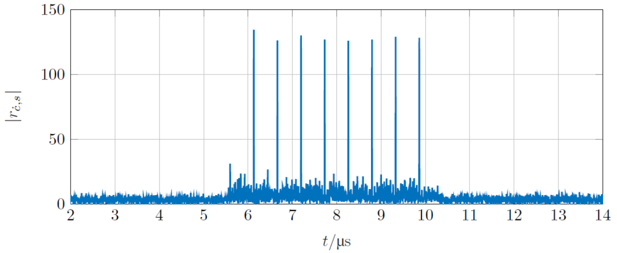
**correlation with PSS**

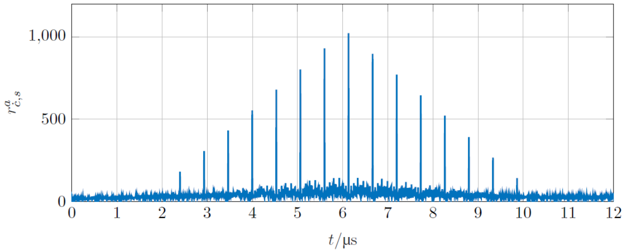
$$r_{c,s}[l] = \sum_{n=0}^{L_c} c[n] s^*[n-l]$$



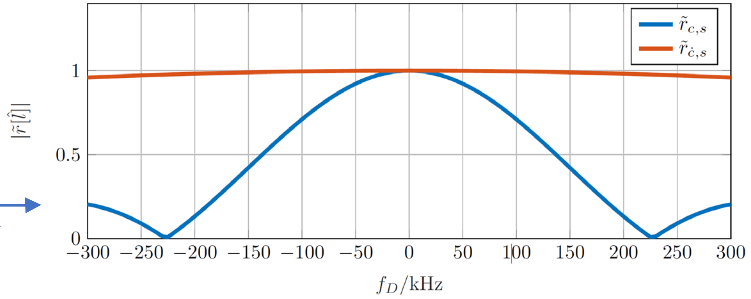

**correlation with subsequence of PSS**

$$r_{\dot{c},s}[l] = \sum_{n=0}^{L_{\dot{c}}} \dot{c}[n] s^*[n-l] \quad \Rightarrow \quad r^a_{\dot{c},s}[l] = \sum_{i=0}^{7} \left| r_{\dot{c},s}[l - iL_{\dot{c}}] \right|$$




correlation peak for unknown Doppler shift




**Higher correlation gain**

**vs.**

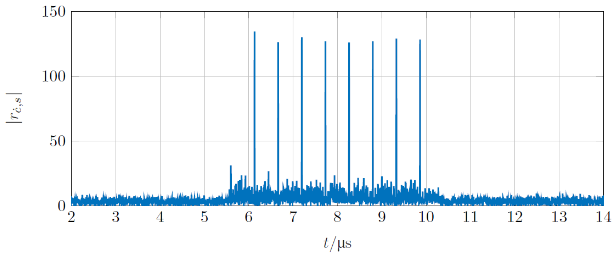
**resilience to Doppler shift misalignment**

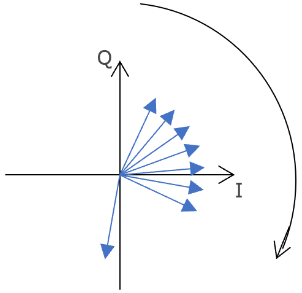
# Frequency estimation



**correlation with subsequence of PSS**

$$r_{\dot{c},s}[l] = \sum_{n=0}^{L_{\dot{c}}} \dot{c}[n] s^*[n-l]$$




Doppler shift causes rotating phase of correlation peaks

**algorithm**

8 correlation peaks

Phase of subsequences as transmitted

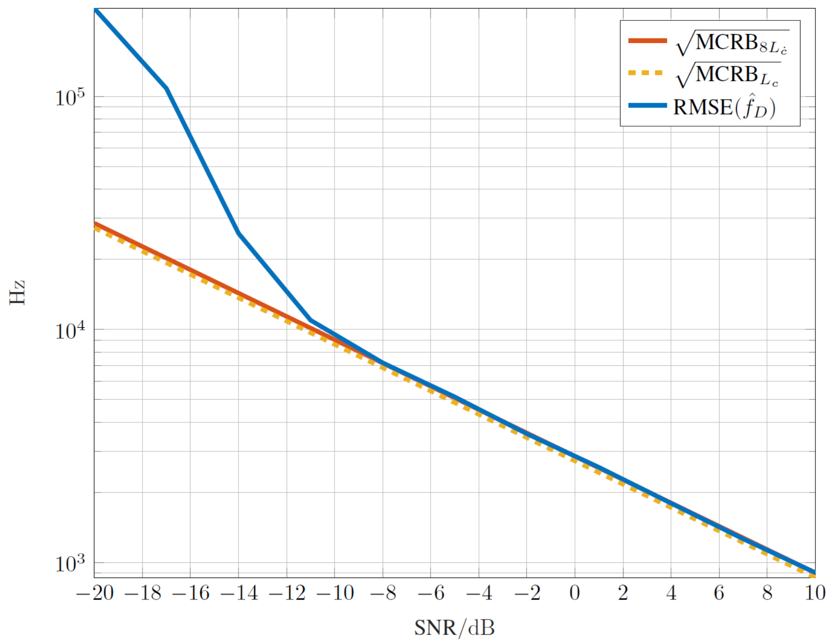
$$z[k] = y[k] \cdot d^*[k]$$

$$R[m] = \frac{1}{L_0 - m} \sum_{k=m}^{L_0-1} z[k] \cdot z^*[k-m]$$

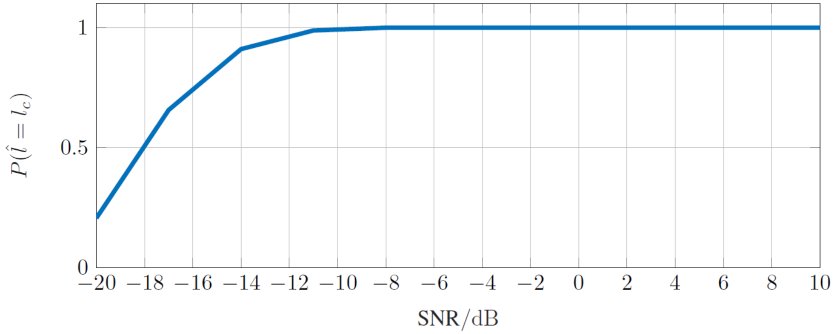
with $1 \leq m \leq L_0 - 1$

$$\hat{f}_D = \frac{1}{5\pi T_s L_{\dot{c}}} \arg\left\{ \sum_{m=1}^{4} R[m] \right\}$$

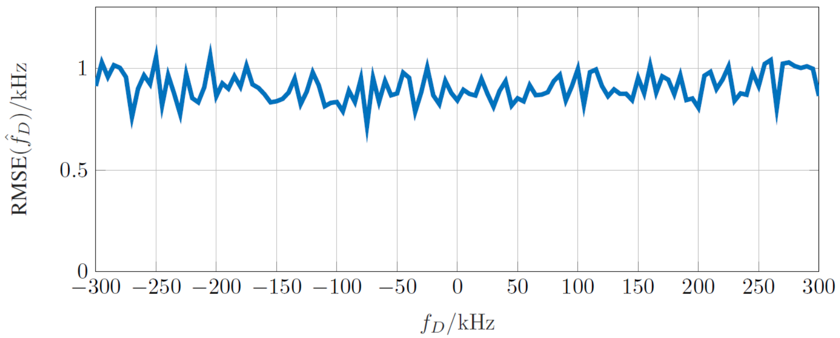
# Analysis



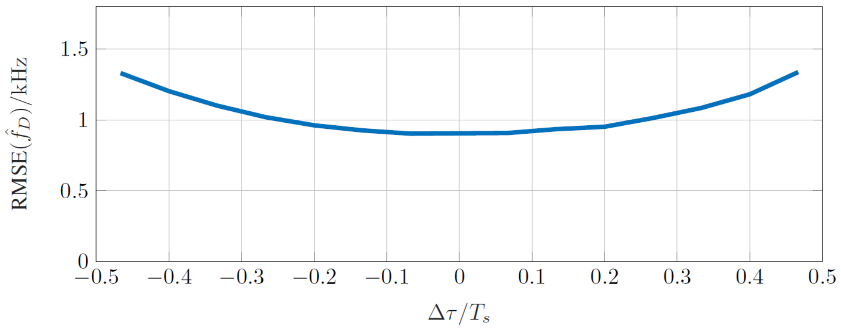

$\mathrm{RMSE}(\hat{f}_D)$ of the Doppler shift estimation for different SNRs



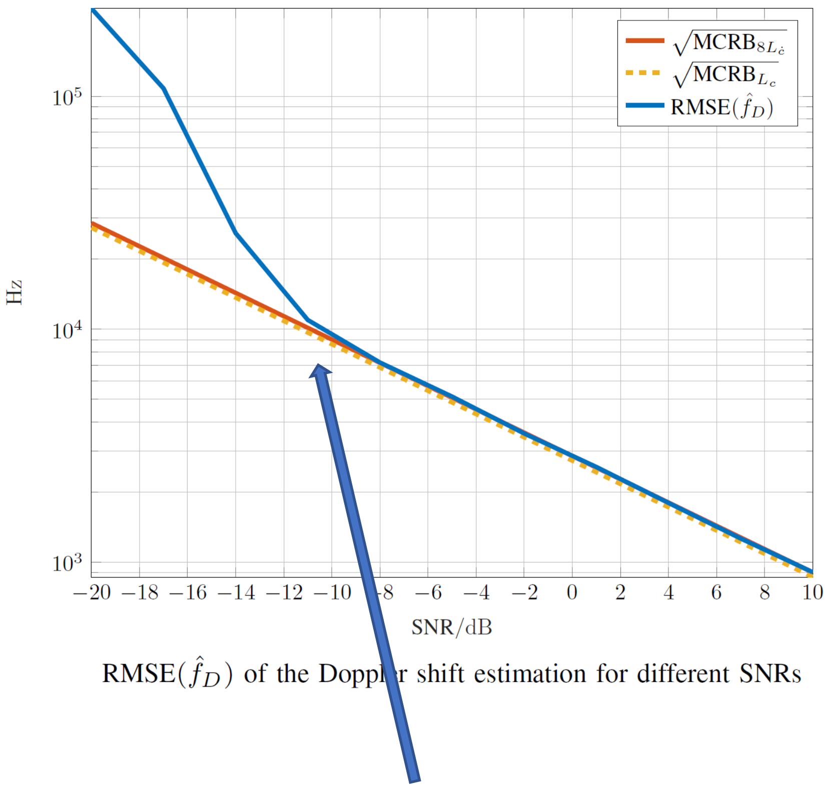

Probability that the PSS is detected correctly




$\mathrm{RMSE}(\hat{f}_D)$ for different Doppler shifts $f_D$ and SNR $= 10\,\mathrm{dB}$




$\mathrm{RMSE}(\hat{f}_D)$ for different sample timing offsets $\Delta\tau$ and SNR $= 10\,\mathrm{dB}$




$\mathrm{RMSE}(\hat{f}_D)$ of the Doppler shift estimation for different SNRs

$$\mathrm{SNR}_T = -11\,\mathrm{dB}$$

- Algorithm works efficiently for

$$\mathrm{SNR}_T \geq -11\,\mathrm{dB}$$

- SNR with isotropic radiator as receiver antenna

$$\mathrm{SNR}_R = -20.9\,\mathrm{dB}$$

- Margin $M$

- Required receiver antenna gain

$$G_R = \mathrm{SNR}_T + M - \mathrm{SNR}_R$$

➡ $G_R \geq 10\,\mathrm{dBi}$ for $M = 0\,\mathrm{dB}$

# Thank you!

Zentrum für Digitalisierungs- und Technologieforschung der Bundeswehr

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NextGenerationEU

W. Stock, R. T. Schwarz, C. A. Hofmann and A. Knopp, "**Survey On Opportunistic PNT With Signals From LEO Communication Satellites**," in IEEE Communications Surveys & Tutorials