Curtin University

Precise Orbit Determination: A Requirement for LEO-PNT Systems

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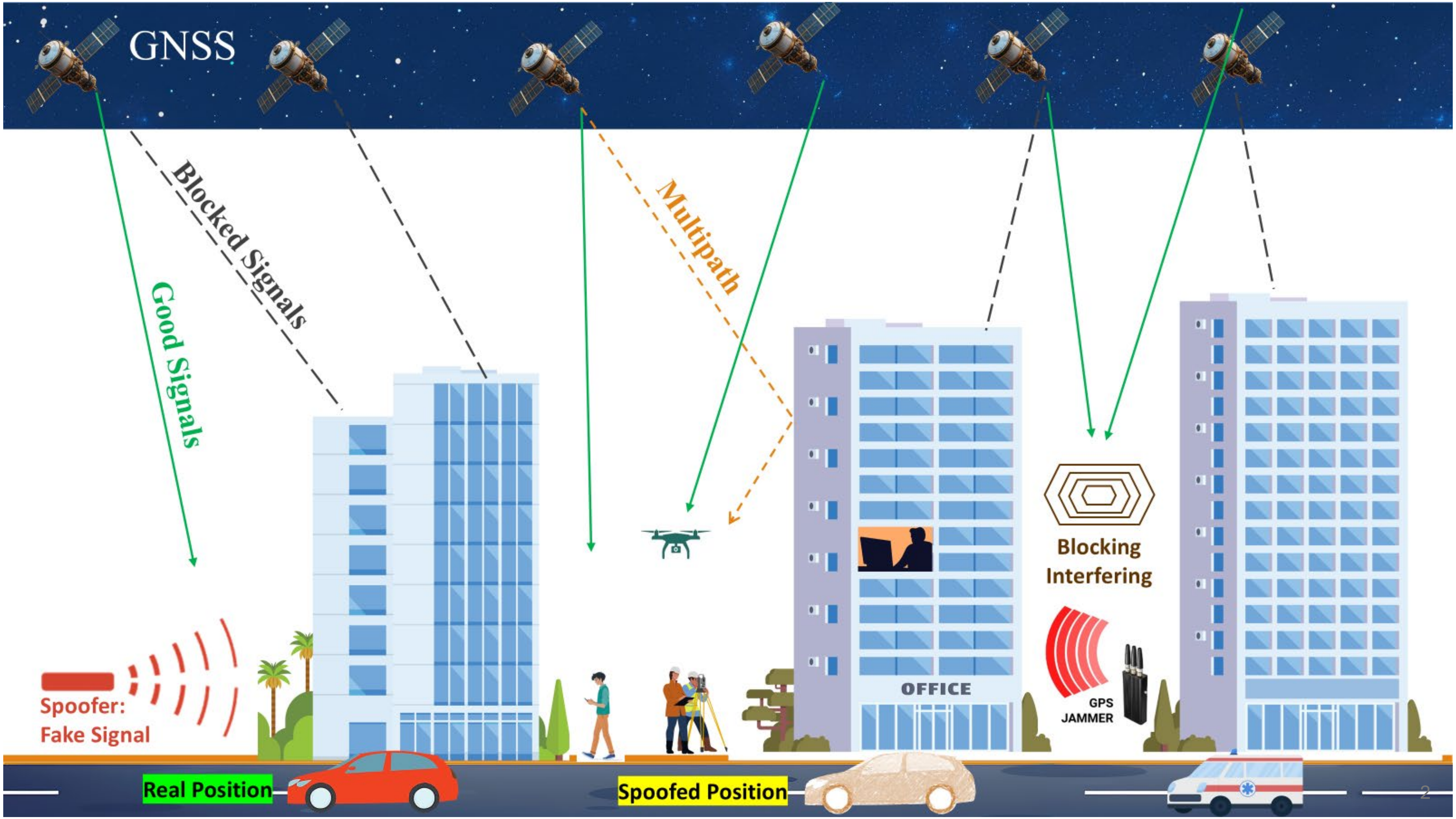
IEEE LEO Sats Workshop – May 2025

GNSS - SPAN

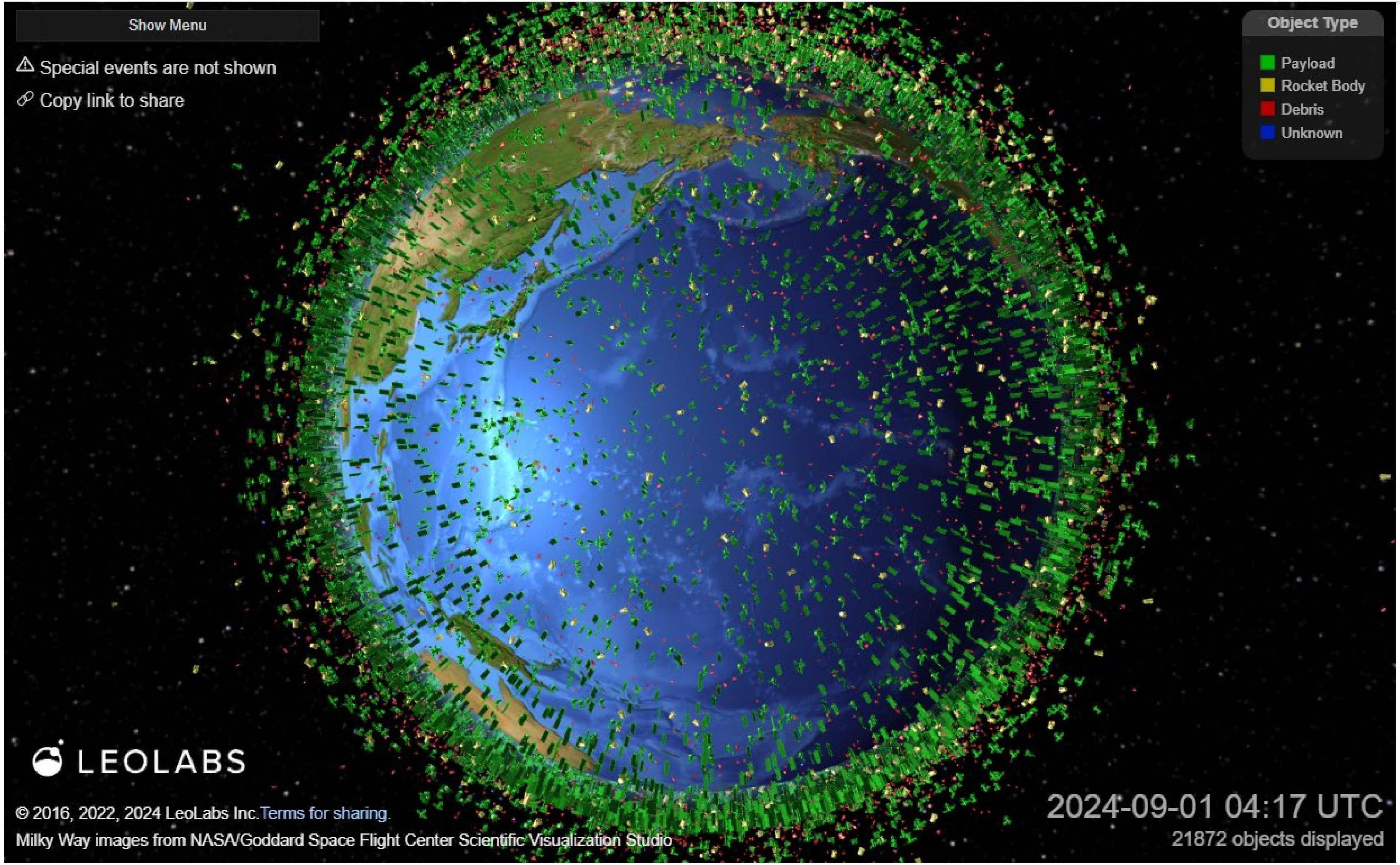
PNT in Challenging Environments (Only GNSS)



What is going on in the low-Earth orbit (LEO) region?

- Space Proliferation!
- LEO region: ~500-1200 km

How many satellites do you think we have in LEO region?



What is going on in low-Earth orbit (LEO) region?

➢ Major LEO Satellite Constellations

Constellation	Company	Current Satellites (est.)	Planned Total	Status
Starlink	SpaceX	**>7,000**	**42,000**	Operational & expanding
OneWeb	OneWeb/Eutelsat	**~630**	**648**	Nearly complete
Kuiper	Amazon	**-**	**>3,000**	Early deployment
GuoWang	China SatNet	**10**	**~13,000**	Early deployment
Iridium NEXT	Iridium	**66**	**66**	Complete
Globalstar	Globalstar	**25**	**25**	Complete

~8,000 Satellites

60,000+ Satellites by 2030

Future of space-based PNT

GNSS Limitations

+

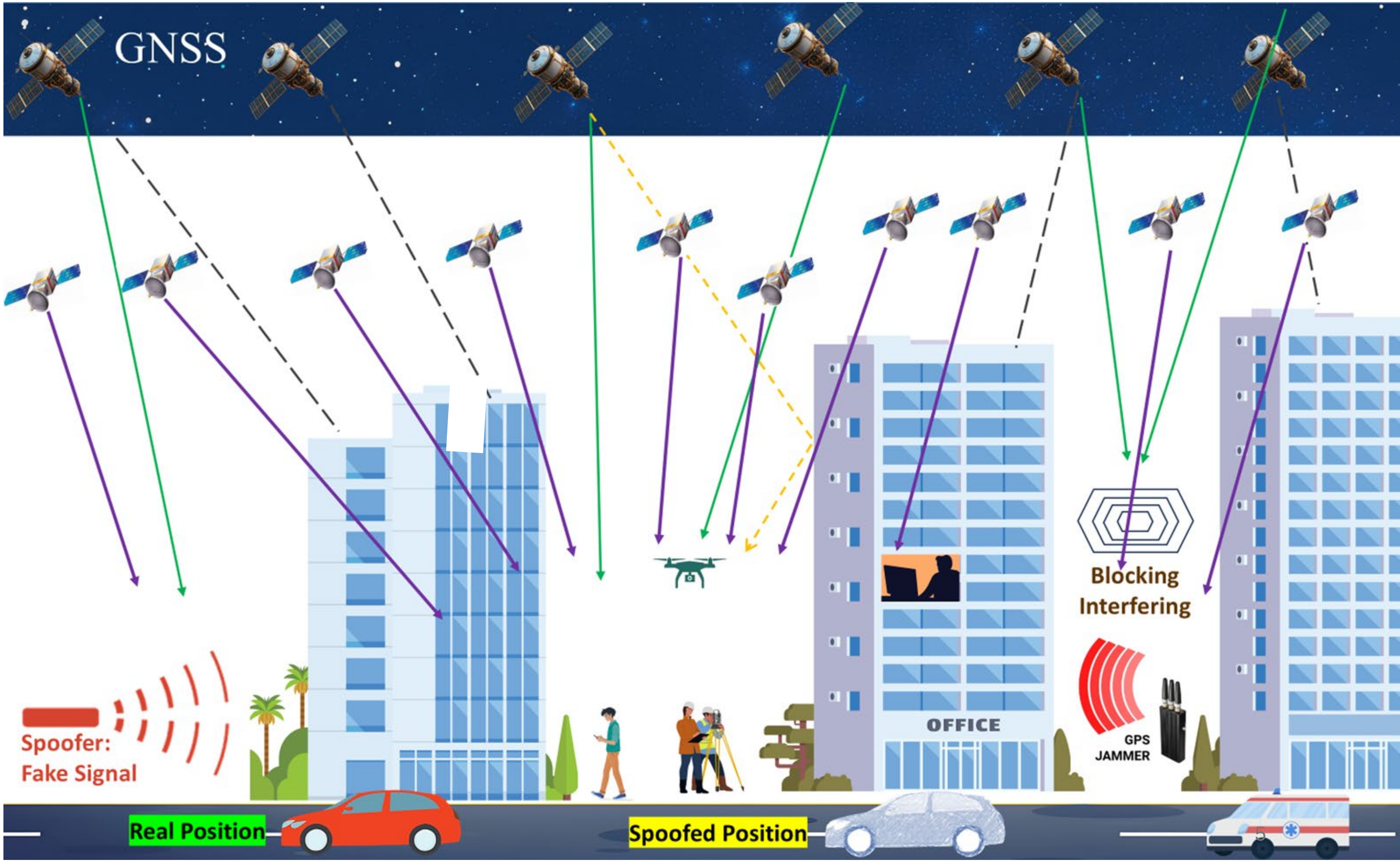
Constellations in LEO (Dedicated for PNT or Broadband)

⇩

LEO-PNT systems

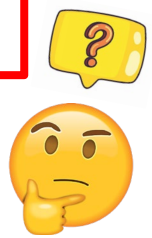
We are developing models for precise PNT for such constellations

(Satellite and User sides)



Let's start with the satellite side models in LEO-PNT systems

It is critical to know the exact positions of LEO satellites in space (in real time!)



- Basics of satellite-based positioning:

$$\text{Pseudorange:} \quad \rho = \sqrt{(X_s - X_P)^2 + (Y_s - Y_P)^2 + (Z_s - Z_P)^2} + c(dt_r - dt^s) + b$$

(Orbit Determination) ←

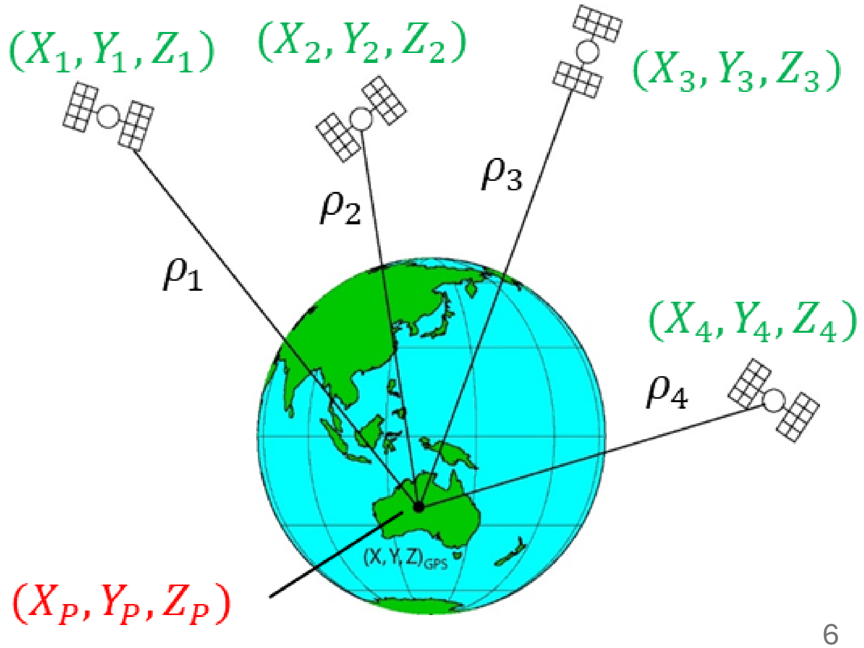
Known: Satellites orbits and clocks (X_s, Y_s, Z_s, dt^s)

Unknown: User positions and clock offset (X_P, Y_P, Z_P, dt_r)

Higher orbital accuracy

↓

Higher positioning accuracy



We need Precise Orbit Determination (POD)

i.e., Orbital accuracy at several cm levels

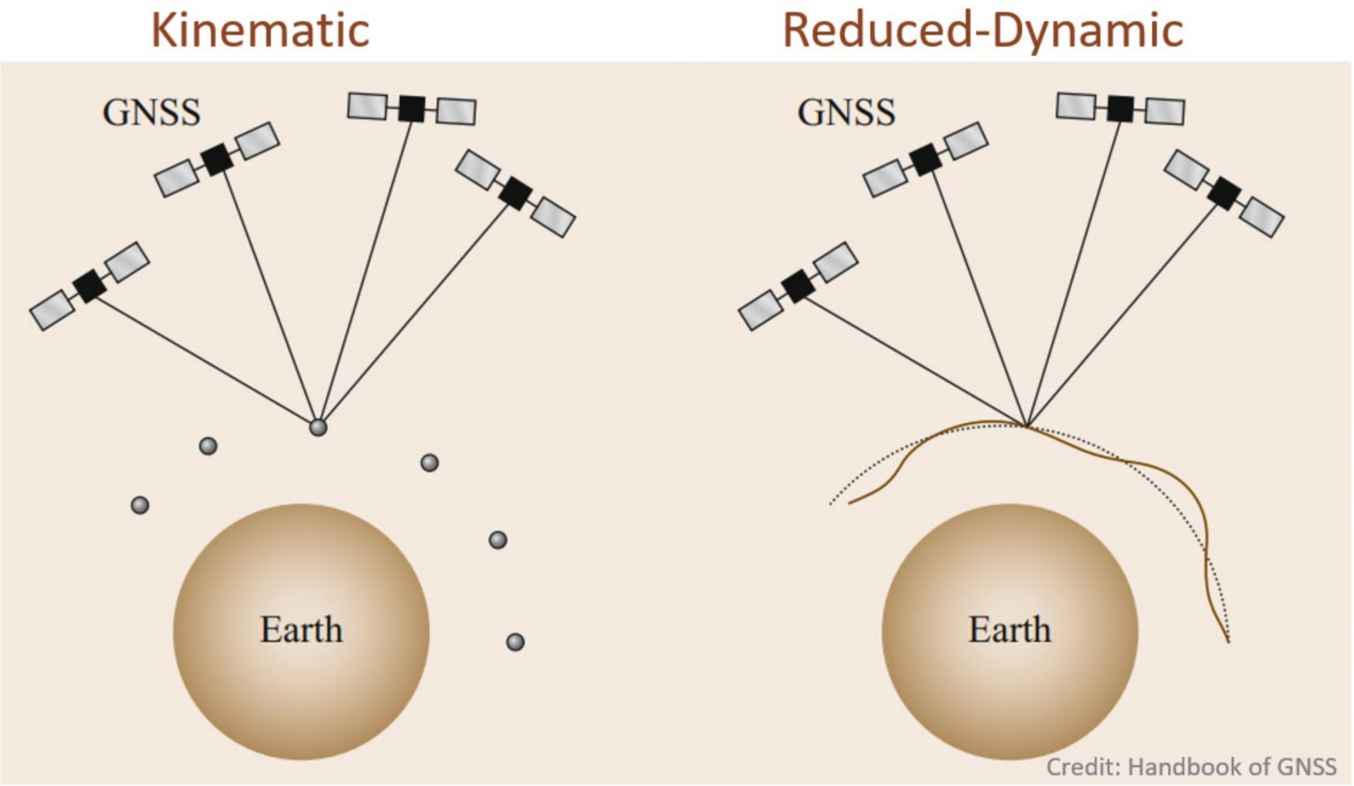
Precise Orbit Determination (POD) of LEO satellites

Kinematic POD:

- Based on Precise Point Positioning (PPP)
- Sensitive to the outliers
- No observation → No orbit
- Bad observation → Low orbital accuracy

Reduced-Dynamic POD

- Based on solving the equation of motion
- Integrating with the GNSS observations
- Estimating stochastic accelerations to compensate for dynamic model deficiencies
- Continuous and more accurate orbit
- Cumbersome processing

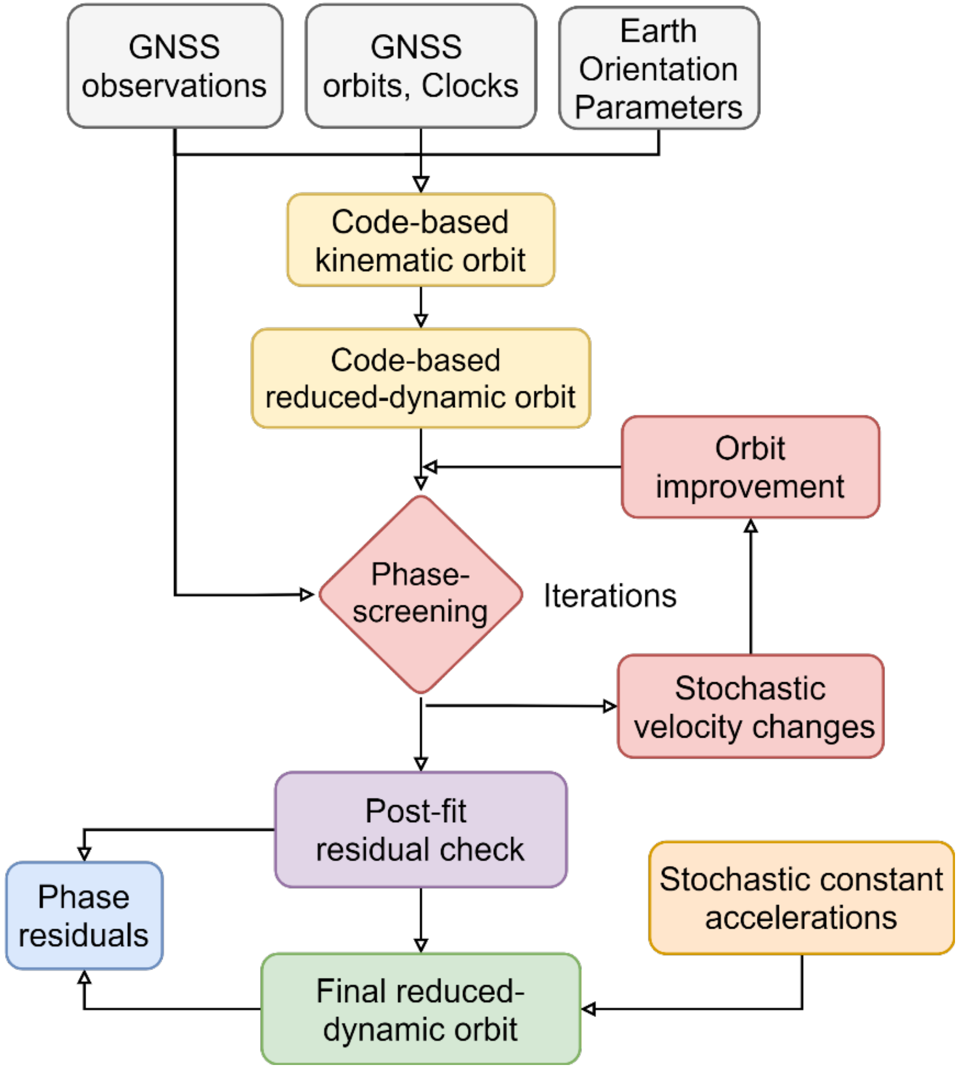


Post-mission POD?

real-time/ onboard POD?

https://doi.org/10.1061/(ASCE)SU.1943-5428.00003

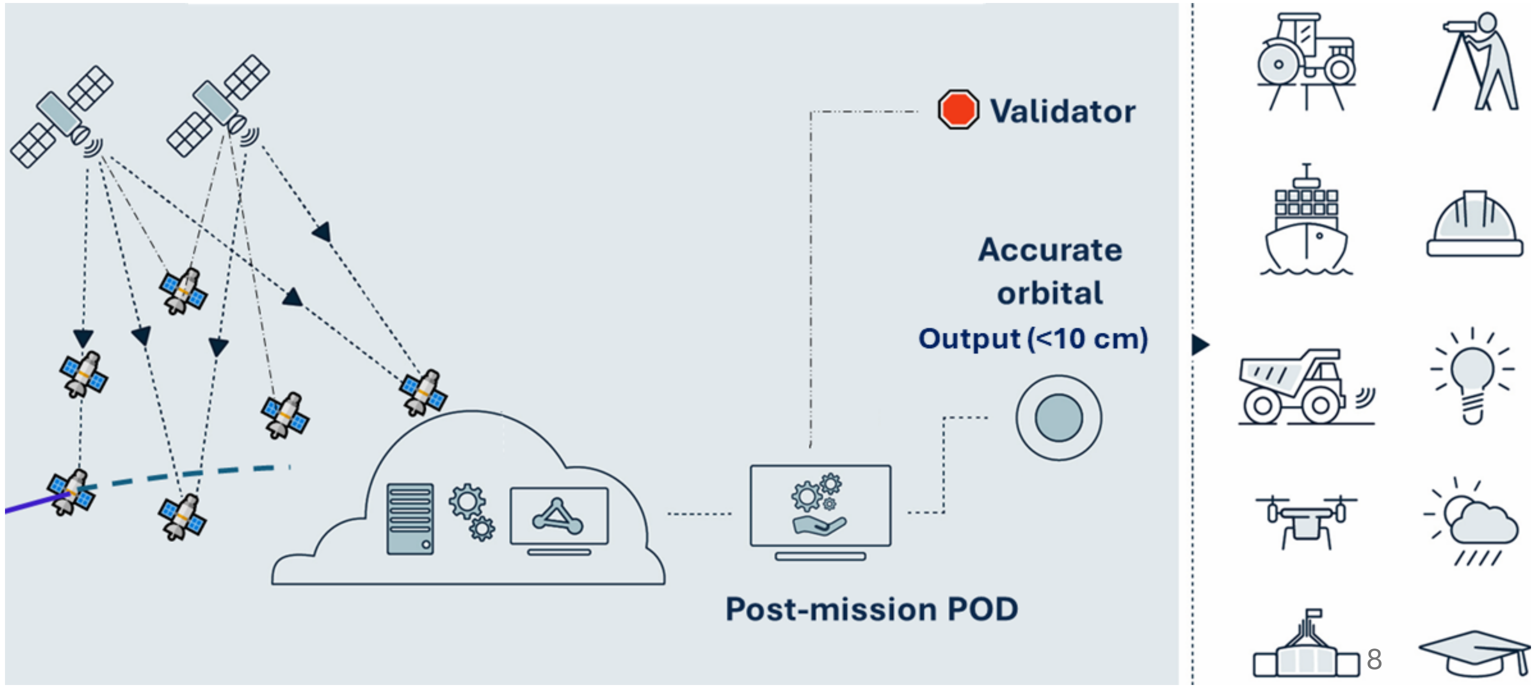
Post-mission POD



Assumption: We have access to the onboard GNSS observation of LEO satellites

- The most accurate POD mode
- Needs to send data to the ground stations
- Users will receive orbits and clocks through the Internet or Space links

(Near Real-Time Output + Orbit and Clock Predictions)

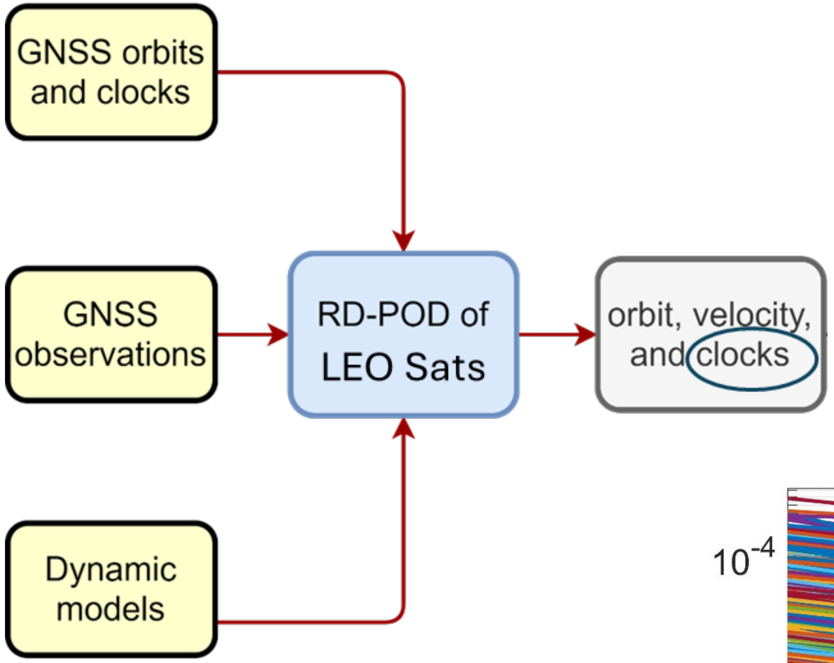


Post-mission POD: Orbital accuracy for various LEO missions

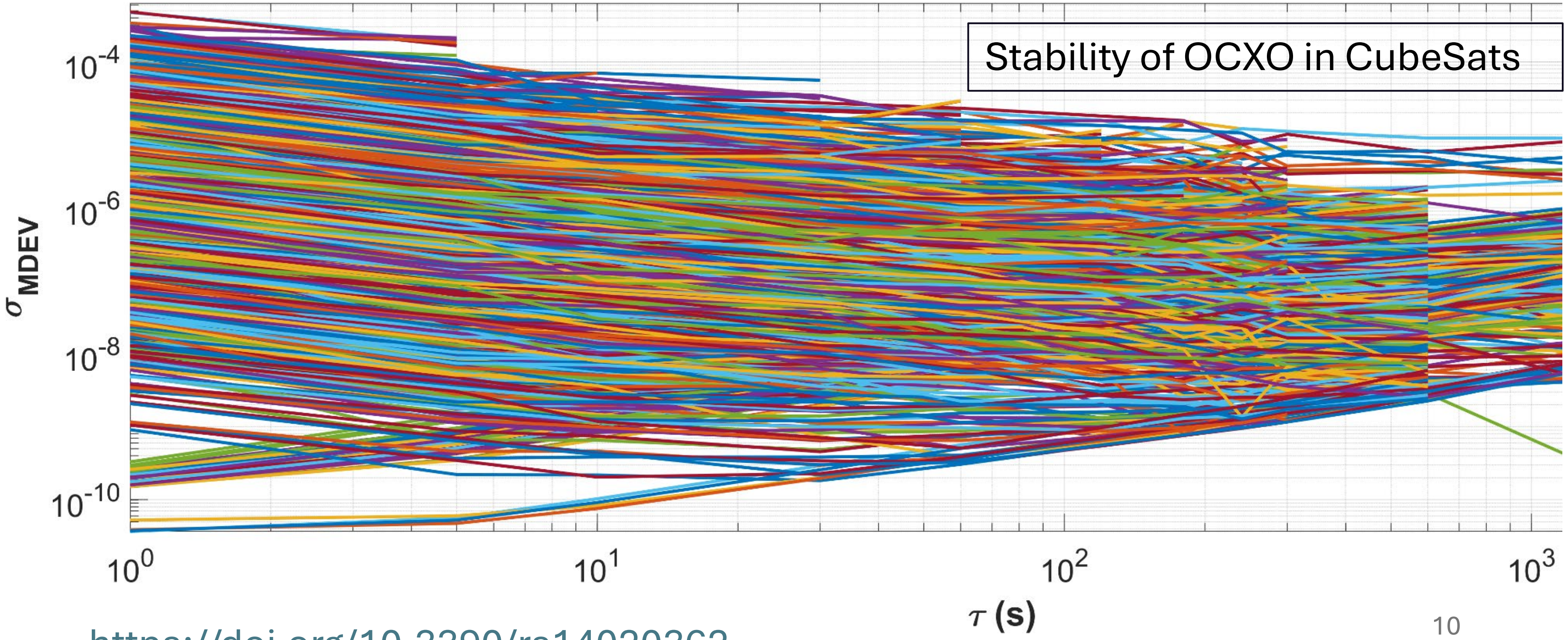
Mission	Accuracy (cm)	Reference
CHAMP	**<2.7**	(Švehla and Rothacher 2005)
GRACE A&B	**<3.0**	(Jäggi et al. 2007)
GOCE	**<2.3**	(Bock et al. 2011)
Swarm-A&B&C	**<5**	(Jäggi et al. 2016) (Van Den Ijssel et al. 2015)
Fengyun-3C	**3.4**	(Li et al. 2017)
TG02	**3.5**	(Li et al. 2018)
Sentinel 1-A&B	**1.5**	(Peter et al. 2017, Fernández et al. 2018, Fernández 2019)
Sentinel 2-A&B		
Sentinel 3-A&B		
GRACE-FO C&D	**1.5**	(Kang et al. 2020)
Spire CubeSats	**<10**	(Allahvirdi-Zade 2023)

https://doi.org/10.1007/s10291-020-01078-8

Post-mission POD: Clock stability issue



If the satellites are not equipped with Ultra Stable Oscillators (Atomic Clocks), the clocks from POD are not stable enough for predictions



MDEV: Modified Allan Deviation
τ: Averaging time

https://doi.org/10.3390/rs14020362

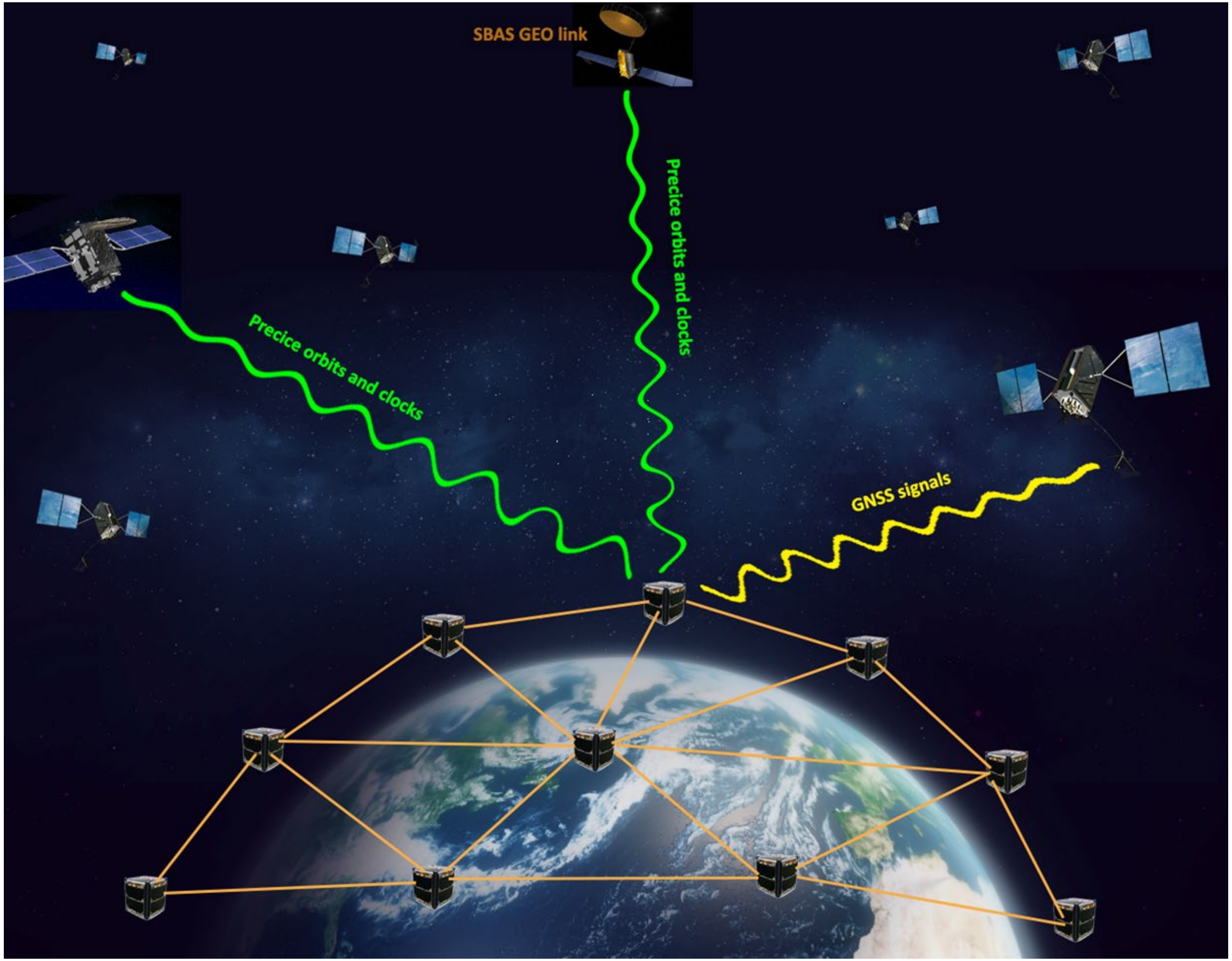
Real-time/onboard LEO POD

Everything is done onboard

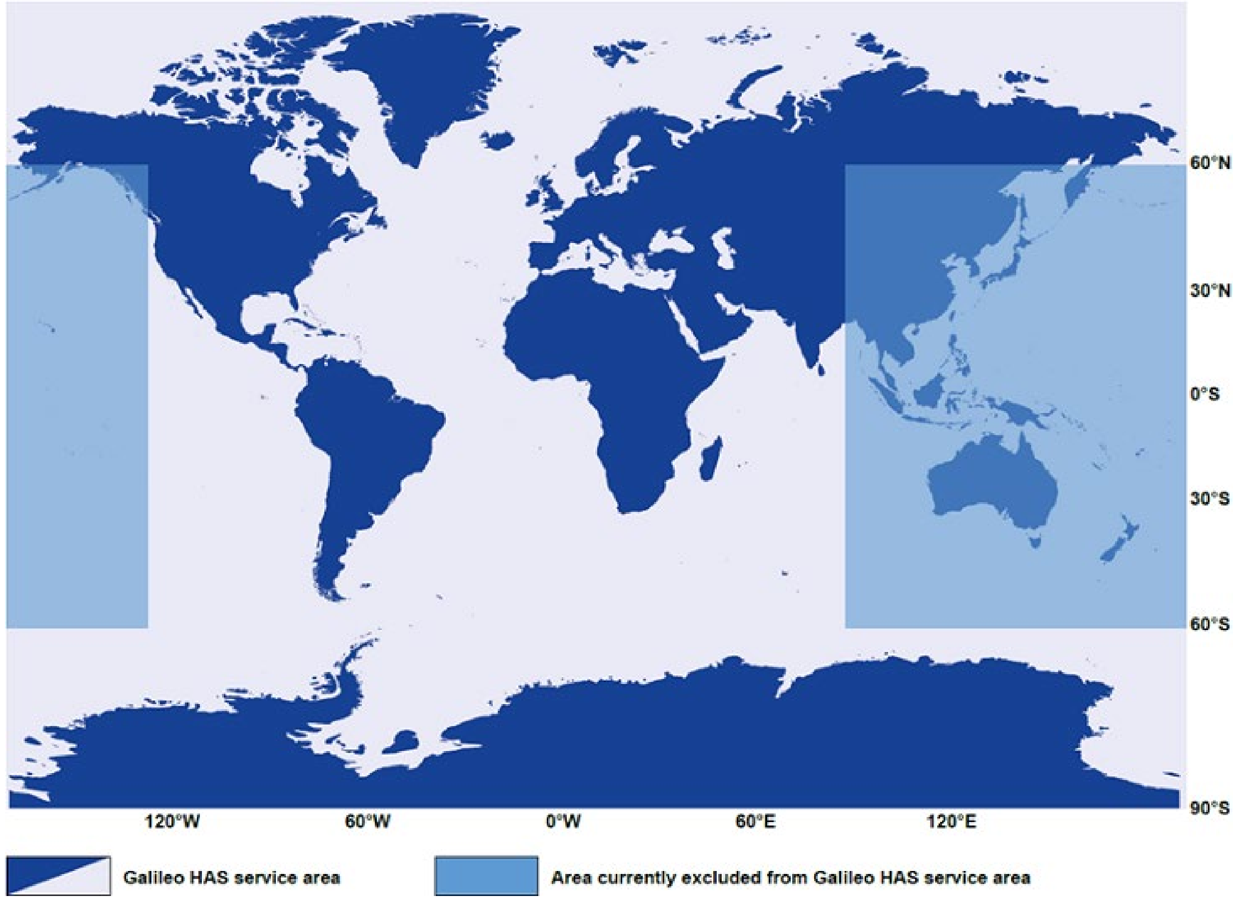
The estimated orbits will be sent to the users as part of the transmitted signals

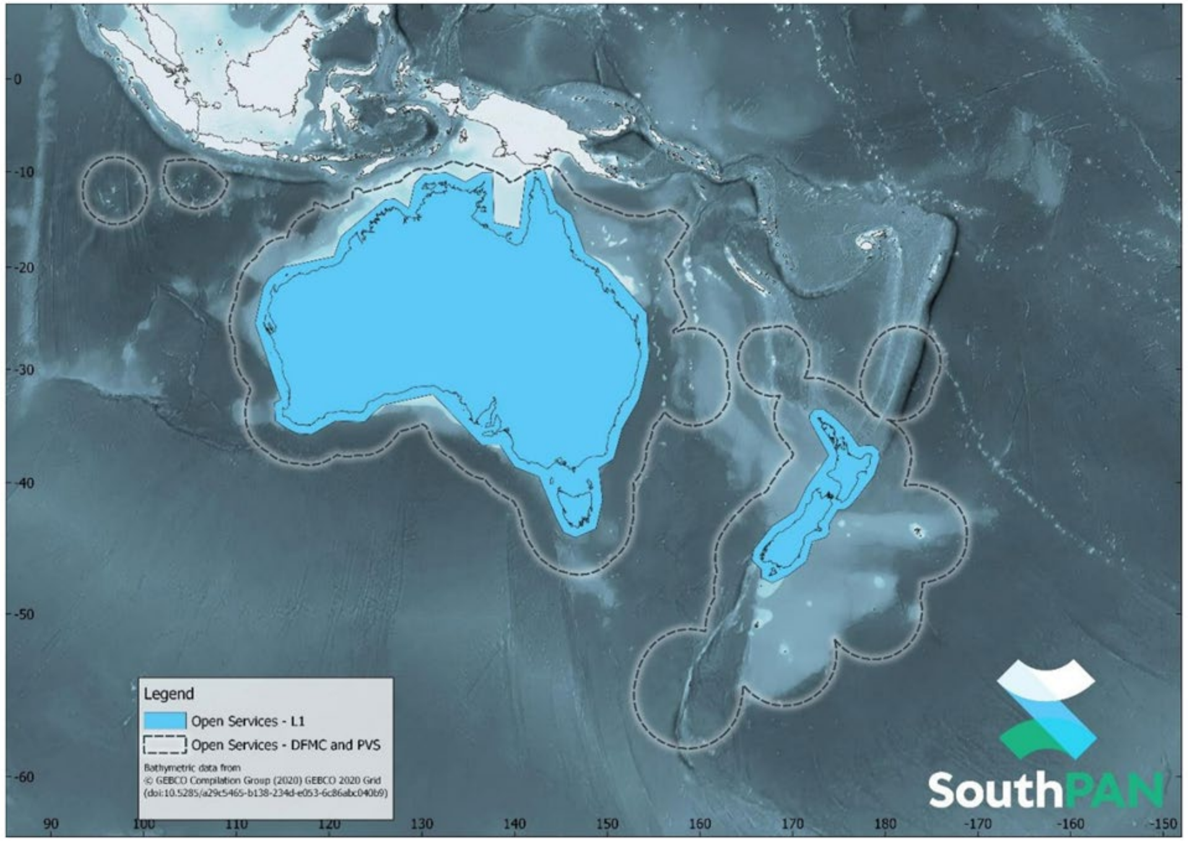
Challenges:

- Needs GNSS corrections in space
- Limited power and processing budget
- Commercial Off-the-Shelf (COTS) Sensors
- Inter-satellite links



Real-time LEO POD: GNSS corrections in space (space links)



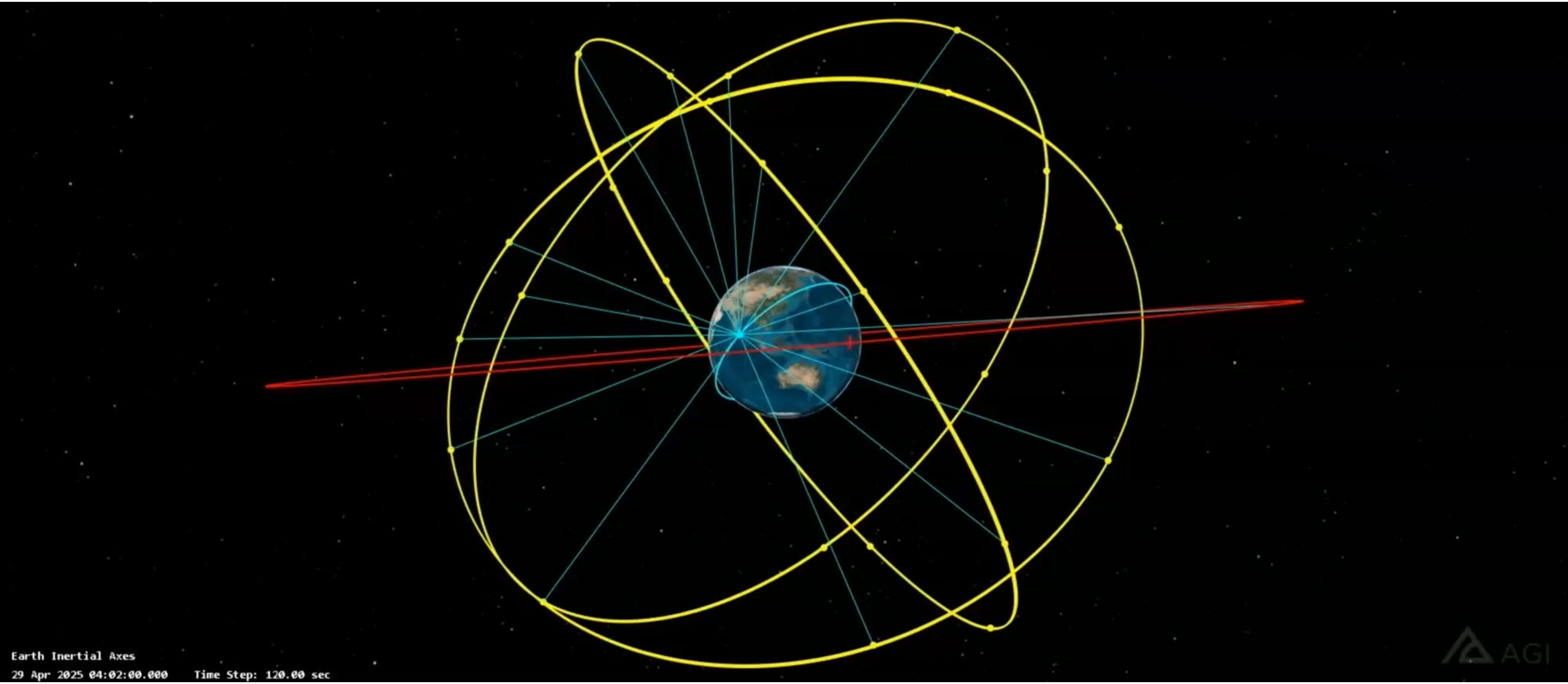


Galileo High Accuracy Service (HAS)

AU/NZ SBAS (SouthPAN)

Other space links: MADOCA (QZSS), Commercial Companies

Real-time LEO POD: GNSS corrections in space



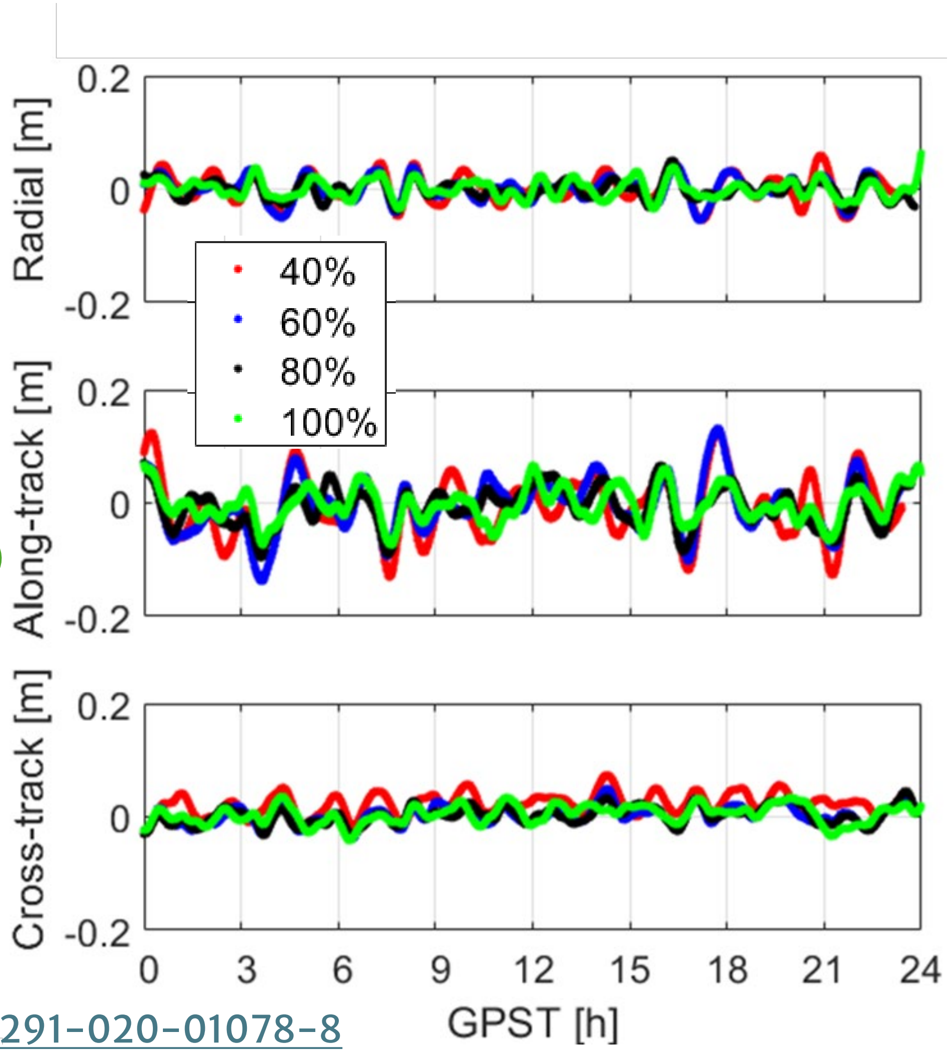
Real-time LEO POD: Limited power and processing budget

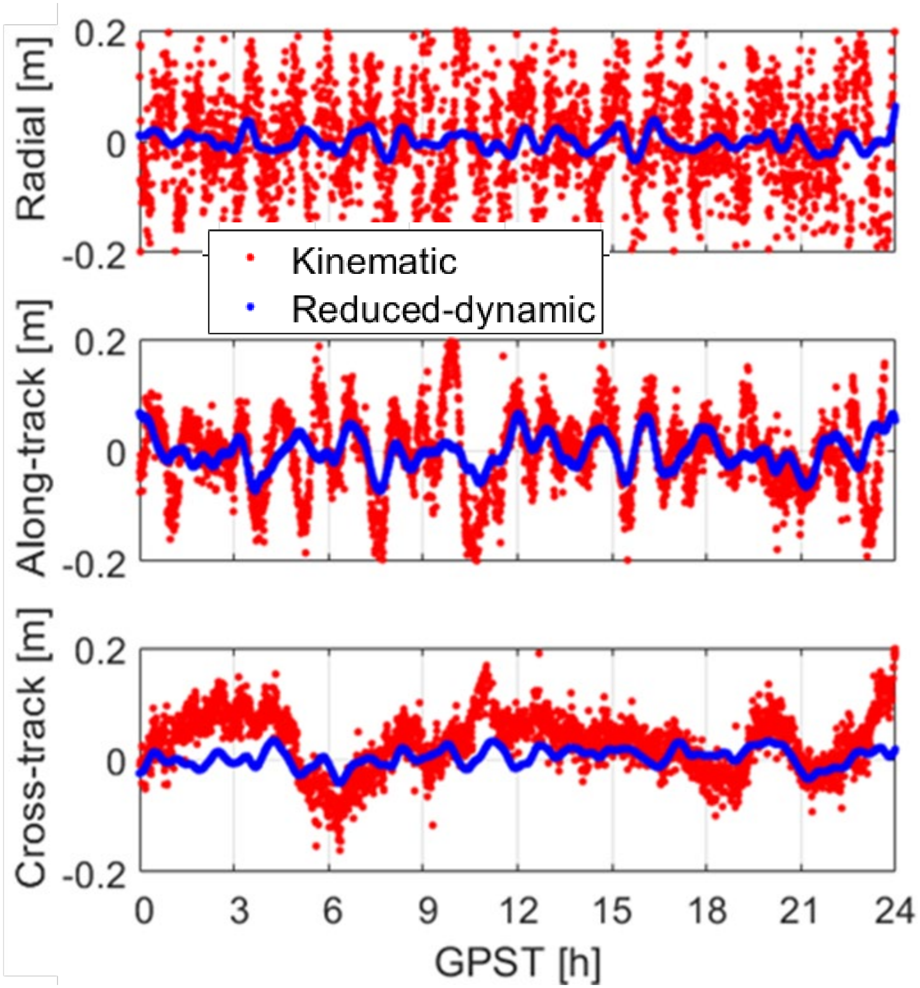
LEO satellites operate under various constraints:

- Applying different duty cycles (%); storing GNSS observations with lower sample intervals.

KIN POD using SouthPAN products

RD POD using SouthPAN products





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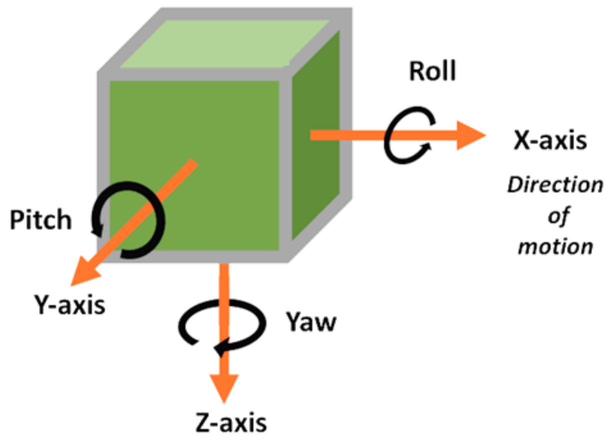
Real-time LEO POD: Commercial Off-the-Shelf (COTS) Sensors

Sensor quality directly impacts orbital accuracy

Example: Attitude determination of a satellite



Magnetometers

- Low accuracy (1°)

Sun Sensor

- Works only in sunlight
- Low accuracy (0.5°)

Star Tracker

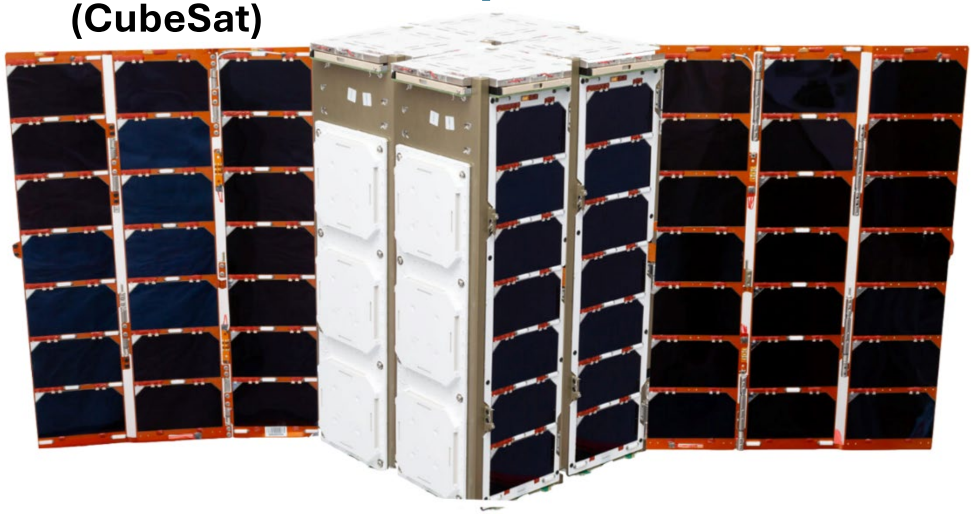
- Price: ~ $45k
- Weight: 275 g

Real-time LEO POD: Limited power and processing budget

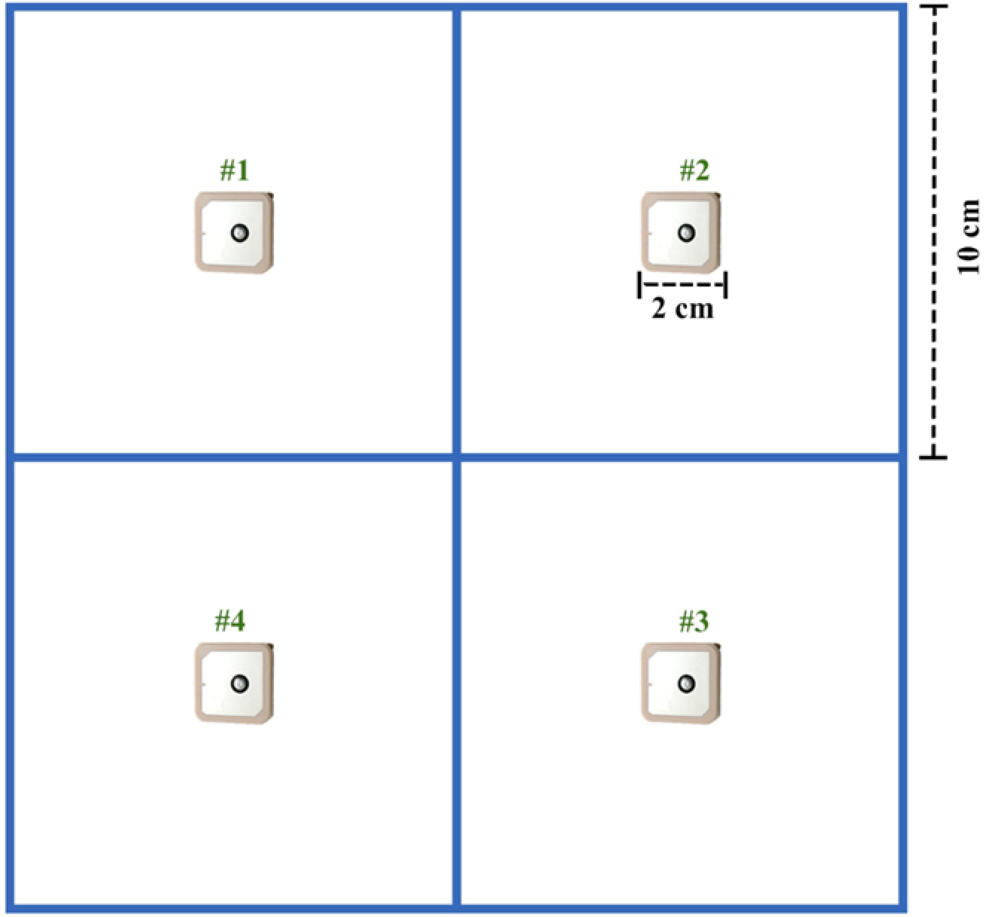
- ✓ **Slighter** than other attitude sensors: ~45 g
- ✓ **Less power** requirement: ~0.1 W
- ✓ **Cheaper**: ~$4K
- ✓ **Using our model, it is more precise!**

We developed models to reach higher accuracy for attitude and orbits of satellites

Small Satellite (CubeSat)



Zoomed To Panel



Real-time LEO POD: Limited power and processing budget

$$E\left(P_r\right)=\left(e_f^T \otimes\left[\left(g_r^{1s}\right)^T,\cdots,\left(g_r^{ms}\right)^T\right]^T\right)b_r+\left(\left[\mu_1,\cdots,\mu_f\right]^T \otimes \mathrm{I}_{m-1}\right)i_r+\varepsilon_{P_r}$$

$$E\left(\Phi_r\right)=\left(e_f^T \otimes\left[\left(g_r^{1s}\right)^T,\cdots,\left(g_r^{ms}\right)^T\right]^T\right)b_r-\left(\left[\mu_1,\cdots,\mu_f\right]^T \otimes \mathrm{I}_{m-1}\right)i_r+\left(diag\left[\lambda_1,\cdots,\lambda_f\right]\otimes \mathrm{I}_{m-1}\right)z_r+\varepsilon_{\Phi_r}$$

Explained in the paper:

Allahvirdi-Zadeh, A., & El-Mowafy, A. (2024). **Array-Aided Precise Orbit and Attitude Determination** of CubeSats using GNSS. *NAVIGATION*: Journal of the Institute of Navigation, 71(3).;

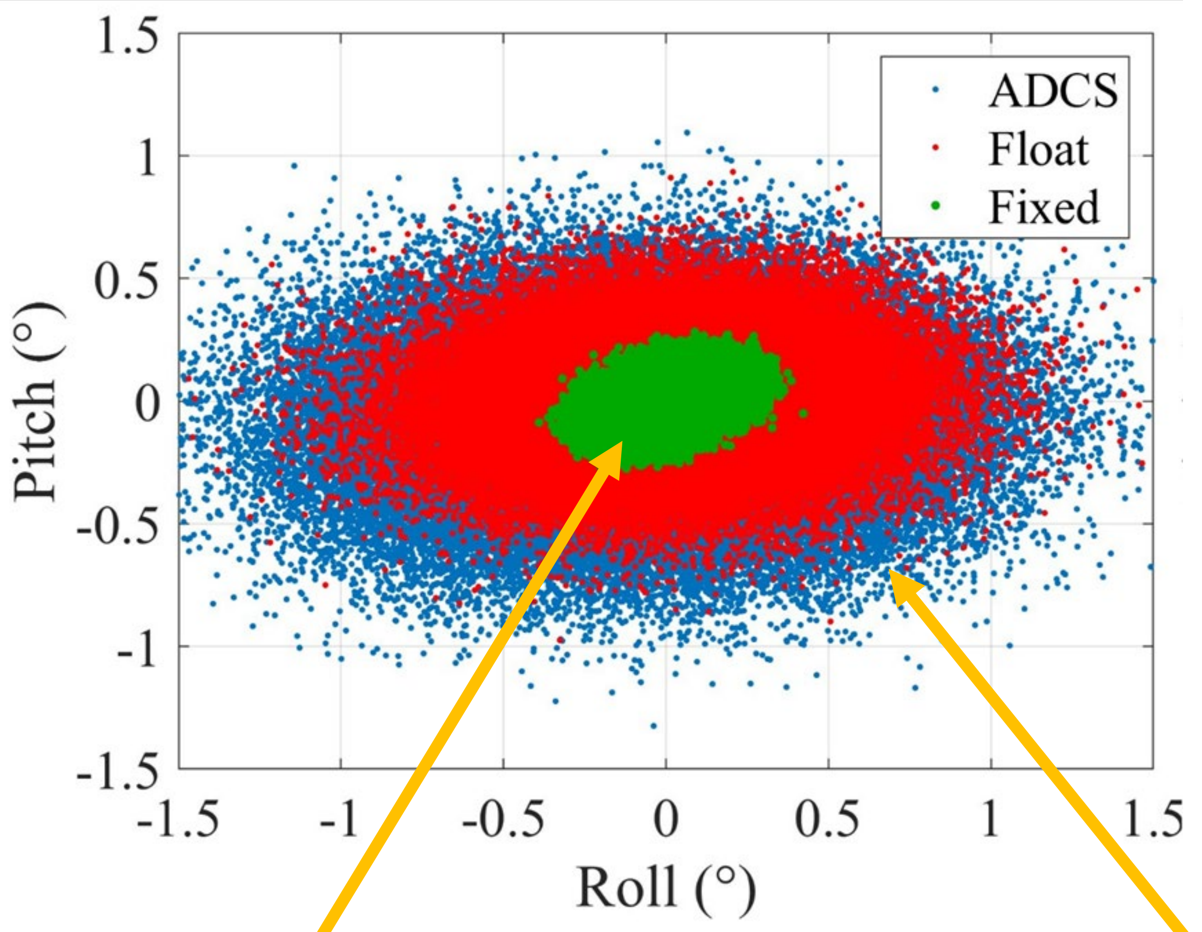
https://doi.org/10.33012/navi.651

$$E\left(vec\left[\bar{y},Y\right]\right)=vec\left(A\left[\bar{z},Z\right]+G\left[\bar{b},RB_0\right]+\left[d_r,0\right]\right),\quad Z\in\mathbb{Z}^{f(m-1)\times(a-1)},\quad R\in O^{3\times q}$$

$$Q_{vec\left[\bar{y},Y\right]}=blkd\left[\left(e_a Q_r^{-1} e_a^T\right)^{-1},D_{a-1}Q_r D_{a-1}^T\right]\otimes\left(Q_f \otimes blkd\left[D_{m-1}Q_p D_{m-1}^T,D_{m-1}Q_\varphi D_{m-1}^T\right]\right)$$

Real-time LEO POD: Limited power and processing budget

Attitude Results



Using our model

Using Magnetometer and sun sensors

Orbit Results

OBSERVATIONS	3D RMSE (M)
ONE ANTENNA	0.118
A-POD WITH 4 ANTENNAE	0.041

✓ **Orbits and attitude of the satellites are improved**

Real-time LEO POD: Inter-satellite links

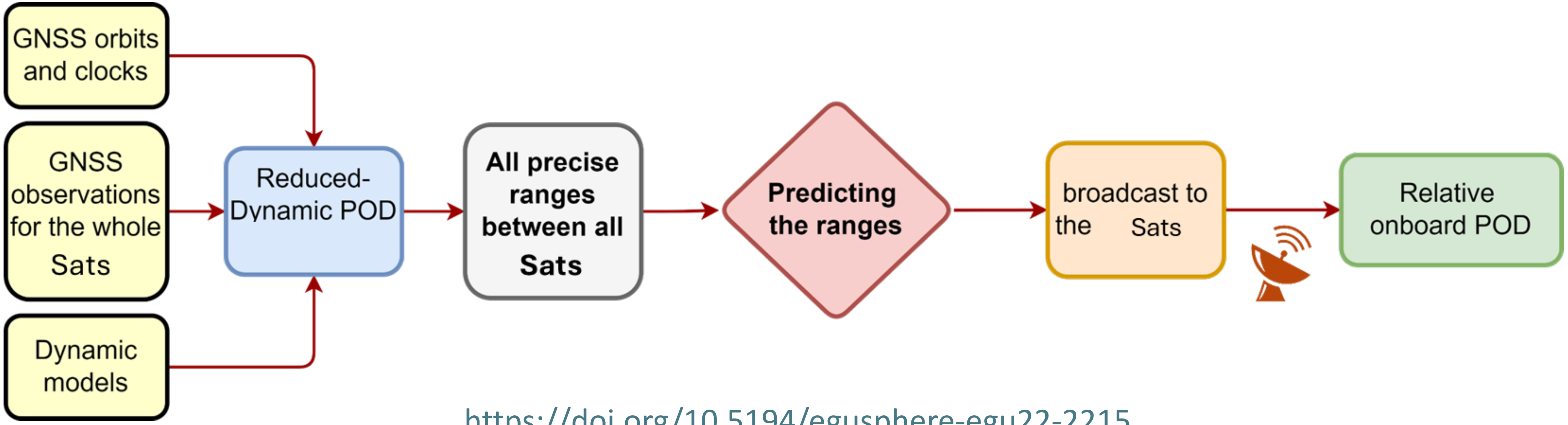
Small satellites face significant challenges in Real-Time LEO POD:

- **Limited power and CPU,**
- **COTS sensors quality,**
- **Unstable oscillator, etc.**

Augmenting POD with precise inter-satellite ranges

- **Meet the power and computational expectations**
- **Reduced the impact of the receiver-dependent errors**
- **Strengthen the model**

Real-Time sub-meter accuracy achievable for CubeSats
Several cm levels are achievable for LEO satellites

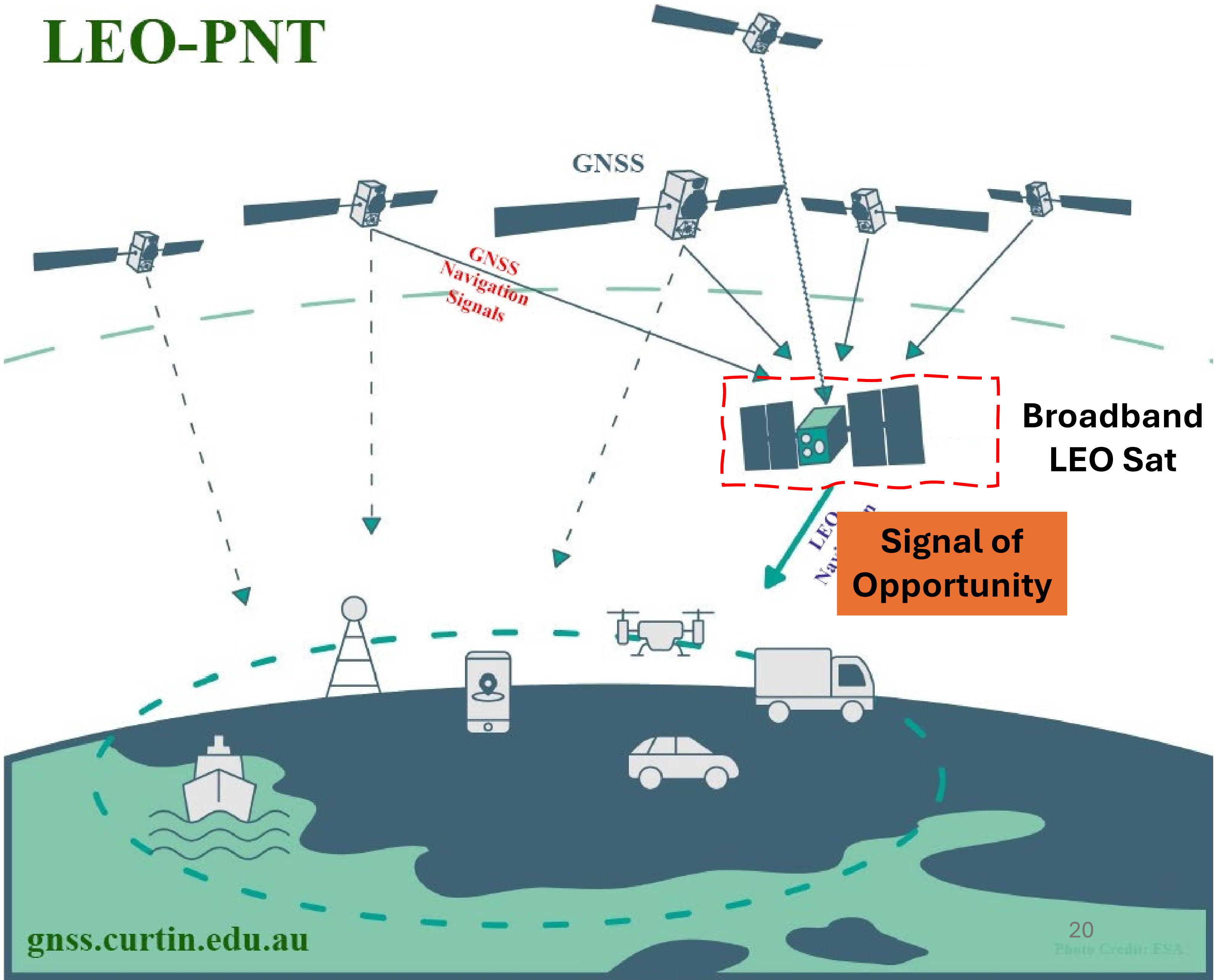


https://doi.org/10.5194/egusphere-egu22-2215

User-side Models for Broadband LEO-PNT Systems

We developed Doppler-shift-based PNT models for the next generation of PNT systems

Multi-frequency Multi-Constellation models for absolute and relative positioning in the urban area



Doppler Shift Multi-frequency Multi-Constellation model (MF-MC)

$$-\lambda_f D_r^s = \left[\begin{array}{l} \hat{e}_r^s(t)\cdot\left(v_r(t) - R\, v^s(t-\tau)\right)\left(\dfrac{1+c\delta\dot{t}^s}{1-\frac{1}{c}\hat{e}_r^s(t)\cdot\left(Rv^s(t-\tau) - \left(\Omega_e\times Rr^s(t-\tau)\right)\right)}\right) \\ + c(\delta\dot{t}_r - \delta\dot{t}^s) + \dot{T}_r^s - \dot{I}_{r,f}^s \end{array} \right](1+\delta\dot{t}_r)^{-1}$$

Explained in the paper:

Allahvirdi-Zadeh, A., & El-Mowafy, A., Wang K., (2025).

Doppler Positioning Using Multi-Constellation LEO

Satellite Broadband Signals as Signals of Opportunity.

NAVIGATION. https://doi.org/10.33012/navi.691

$$u_r = e_f^T \otimes \left[\begin{array}{l} \hat{e}_r^1\cdot(v_r - Rv^1)\left(\dfrac{1+c\delta\dot{t}^1}{1-\frac{1}{c}\hat{e}_r^1\cdot\left(Rv^1 - (\Omega_e\times Rr^1)\right)}\right), \dots, \\ \hat{e}_r^m\cdot(v_r - Rv^m)\left(\dfrac{1+c\delta\dot{t}^m}{1-\frac{1}{c}\hat{e}_r^m\cdot\left(Rv^m - (\Omega_e\times Rr^m)\right)}\right) \end{array} \right]^T (1+\delta\dot{t}_r)^{-1}$$

$$\delta\dot{t} = e_f^T\otimes[c(\delta\dot{t}_r - \delta\dot{t}^1), \dots, c(\delta\dot{t}_r - \delta\dot{t}^m)]^T(1+\delta\dot{t}_r)^{-1}$$

$$\dot{T}_r = e_f^T\otimes\left[\dot{T}_r^1, \dots, \dot{T}_r^m\right]^T(1+\delta\dot{t}_r)^{-1}$$

$$\dot{I}_r = \left[\dot{I}_{r,1}^T, \dots, \dot{I}_{r,f}^T\right]^T \text{ where } \dot{I}_{r,f} = \left[\dot{I}_{r,f}^1, \dots, \dot{I}_{r,f}^m\right](1+\delta\dot{t}_r)^{-1}$$

$$\varepsilon_{D_r} = e_f^T\otimes\left[\varepsilon_{D_r^1}, \dots, \varepsilon_{D_r^m}\right]^T$$

Absolute Mode:

$$E(D_r) = u_r + \delta\dot{t} + \dot{T}_r - \dot{I}_r$$
$$Q_{D_r} = diag\left(\varepsilon_{D_r}\right)$$

Differential Mode:

$$E(\Delta D) = \Delta u = \left({}_{r_1}^{s_2}u - {}_{r_2}^{s_2}u\right) - \left({}_{r_1}^{s_1}u - {}_{r_2}^{s_1}u\right)$$
$$Q_{\Delta D} = diag(\varepsilon_{\Delta D})$$

Limitations of Broadband Signals

1) Broadband Signals are not continuous
2) Signals are unavailable in low elevation angles
3) There are some restrictions on sending signals from more than one satellite to the user in each cell
4) There is no multi-constellation receiver

We simulate Starlink, OneWeb, and Iridium constellations

Positioning results: Positioning using ONLY Broadband Constellations

RECEIVER	LIM. TYPE	X RMSE (M)	Y RMSE (M)	Z RMSE (M)	3D RMSE (M)
Absolute Mode	No limit	1.92	1.66	2.84	3.81
	Elv Angle	2.19	1.33	3.05	3.98
	Elv + Burst	2.53	2.63	3.15	4.83

RECEIVER	LIM. TYPE	X RMSE (M)	Y RMSE (M)	Z RMSE (M)	3D RMSE (M)
Diff. Mode	*No limit*	0.30	0.23	0.28	0.47
	Elv Angle	0.32	0.29	0.30	0.53
	Elv + Burst	0.34	0.38	0.34	0.62

We developed the Network-based precise orbit determination of broadband LEO satellites using Doppler-shift measurements

✓ **Evaluations confirmed the decimetre-level of positioning using the MF-MC model**

If:

- **The satellite orbits are at the sub-dm level of accuracy**
- **More than one satellite is trackable by a user**
- **If a multi-constellation receiver is available**

Conclusions

- Without LEO POD, we cannot expect high accuracy from LEO-PNT systems
- LEO POD is highly affected by the satellite constraints, models, processing budget, and sensor quality
- Mega constellations have different challenges for real-time POD in orbit/clock estimation and predictions
- Scientists are developing the models and overcoming the challenges.
- There are many open areas to research (especially in the POD of mega constellations).

Remaining question: Will constellation owners recognize the opportunity to expand their services and their revenue streams?

Thanks for your attention

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